

Design Optimization of Functionally Graded Concrete Beams

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Abstract

As concrete construction contributes significantly to global pollution, it has become increasingly important to develop sustainable structural systems. Functionally Graded Concrete (FGC) components, produced by embedding hollow spheres within the concrete volume, offer a promising solution by reducing material consumption while maintaining structural performance. This material efficiency can be further enhanced by applying prestressing. However, determining the optimal placement of the spheres in the beam, their radii, as well as the prestressing level to apply leads to a high-dimensional, non-convex and non-smooth problem. This paper presents a latent-space formulation combined with a derivative-free optimization approach for the structural design of such FGC beams. In particular, it introduces a piecewise smooth mapping from an unconstrained latent vector to physical design variables, which enforces bounded radii and a well-defined spacing by construction with remaining geometric and mechanical feasibility conditions being handled via penalties. The resulting optimization problem is solved using a Covariance Matrix Adaptation Evolution Strategy (CMA-ES) with annealed penalties. The benchmark demonstrates that CMA-ES consistently outperforms the competing methods in terms of solution quality and robustness and obtaining approximately 50% concrete mass reduction while satisfying bending, shear, concrete tensile stress, and geometric constraints.

Keywords: Structural optimization, Functionally graded concrete, Evolutionary strategies, Structural design, Low-carbon concrete construction

1. Introduction

Concrete is one of the most widely used construction materials in the world, with around 30 billion tons used annually. Per-capita consumption has tripled over the past four decades. The cement industry alone is responsible for at least 8 % of global anthropogenic CO_2 emissions [1]. The Functionally Graded Concrete (FGC) technology, developed at the University of Stuttgart [2], is a possible answer to these challenges, as it allows to reduce concrete mass consumption significantly while maintaining structural and functional requirements. In particular, concrete hollow spheres are placed inside the components, enabling up to 50% mass savings. An example of a simply-supported FGC slab with a span width of 6 m is illustrated in Fig. 1. [3] [4]

As part of a research project in the Cluster of Excellence IntCDC (Integrative Computational Design and Construction for Transformative Architecture) basalt fiber-reinforced polymeric (BFRP) tendons are used to increase the stiffness through post-tensioning of FGC components [5]. These BFRP tendons are installed in an unbonded manner and guided inside a mineral duct (casting tube), such as depicted in Fig. 2. Such a system not only enables insensitivity to corrosion, it is also much lighter than steel, and enables high-purity recycling and simplified strengthening during the component's life as well as separability of reinforcement and concrete at end-of-life. [6]



Figure 1: Functionally graded concrete beam.
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(a) Side view

(b) Detail view

Figure 2: Embedded mineral duct made of a basalt fiber-reinforced ceramic composite.
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Designing such components, however, leads to a high-dimensional structural optimization problem: the placement of hollow spheres, their number, their radii and the prestressing strategy must collectively satisfy nonlinear bending, shear and tensile stress constraints, as well as collision avoidance and geometric feasibility. The resulting search space is non-convex, non-smooth, and strongly constraint-dominated, making the problem particularly challenging for classical gradient-based approaches.

Exploring beyond classical gradient-based approaches, many works on structural and topology optimization focus on evolutionary algorithms such as genetic algorithms, particle swarm optimization, and evolutionary structural optimization (ESO/BESO). This is particularly the case when design spaces are highly non-convex and constraint-dominated. Recent work has shown that Covariance Matrix Adaptation Evolution Strategy (CMA-ES) is especially effective for such problems. For instance, this strategy has been successfully applied to truss design, structural topology optimization, and finite-element model updating of civil structures [7]. At the algorithmic level, CMA-ES is a stochastic, derivative-free evolution strategy which iteratively samples candidate solutions from a multivariate Gaussian distribution and adapts its mean and covariance based on the selection of samples. This sampling and covariance adaptation process makes CMA-ES particularly suitable for problems in which the objective is rugged and non-convex due to penalty terms and feasibility boundaries, and where the search space is ill-conditioned or non-separable [8].

In this work, a latent-space formulation is combined with CMA-ES to optimize FGC beam designs. A piecewise smooth mapping transforms an unconstrained latent vector into physical design variables, while mechanical and geometric constraints are handled via penalty terms. The penalty weight is increased over the CMA-ES iterations to guide the population toward feasibility without limiting exploration too early. A benchmark against two gradient-based approaches, Adam and Adam + Bayesian Optimization, as well as two evolutionary algorithms, Differential Evolution (DE) and Particle Swarm Optimization (PSO), demonstrates that CMA-ES achieves superior objective values, better robustness and perfect feasibility across all runs.

2. Design constraints

In order to determine the required mechanical properties of the post-tensioned FGC beam, a 4-point bending test problem is considered. From this, the calculations of the tensile stress, bending and shear capacity constraints are derived. The problem is limited to a distribution along the beam's x axis.

2.1. 4-point bending test constraints

In a 4-point bending test, the bending moment can be modeled as

$$M = F a, \tag{1}$$

where F is the applied force in N and a is the distance load initiation to support in mm (see Fig. 3).

The resulting concrete tensile stress in the uncracked state during a 4-point bending test gives

$$\sigma_t = \frac{(M - P e) z_s}{I_y} - \frac{P}{A}, \quad (2)$$

with σ_t the concrete tensile stress in N/mm², M is the bending moment in N mm, e is the eccentricity in mm such that $e = \frac{H}{2} - 0.5 \times 45$ mm, I_y the inertial moment of a component in mm⁴, P the prestress force in N, z_s the distance from the neutral axis to the point where the tensile stress is evaluated in mm and A the effective concrete cross-sectional area (i.e. rectangle beam area minus the circular void) in mm² (see Fig. 3 and Fig. 4). The concrete beam cracks when :

$$\sigma_t > f_{ctm}, \quad (3)$$

where f_{ctm} is the mean tensile strength of concrete in N/mm².

Shear resistance F_{shear} can be modeled in a simplified manner based on [9] as

$$F_{shear} = 0.0525 k^{3/2} f_{ck}^{1/2} W d \alpha_{HS}, \quad (4)$$

where W is the width of a single-sphere unit (see Fig. 3) and d is defined as the effective depth in mm, such that $d \approx 0.9H$ with H the height of the component, f_{ck} is the concrete compression stress in N/mm², k and α_{HS} are geometry-based factors evaluated per sphere location and defined as :

$$\alpha_{HS} = \frac{\sqrt{2} W H - \pi r_i^2}{\sqrt{2} W H}, \quad (5)$$

where r_i is the radius of the sphere and

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2. \quad (6)$$

The shear resistance should verify the following inequality :

$$F_{shear} > F_{demand}, \quad (7)$$

where the shear demand F_{demand} along the beam is represented by the piecewise function

$$F_{demand}(x) = \begin{cases} F_{ult}, & \text{for } x \text{ in the outer regions.} \\ 0, & \text{for } x \text{ in the inner region.} \end{cases} \quad (8)$$

where F_{ult} is the applied shear demand level (magnitude of shear force in the shear spans), see Fig. 4.

In addition to stress and shear checks, the required reinforcement area in the tensile zone, denoted A_s , is limited by a maximum reinforcement ratio of 2%. This area is estimated as

$$A_s = \frac{1}{f_{yk}} \left(\frac{F_{ult} a}{z} - P \right) \quad (9)$$

where f_{yk} is the characteristic yield strength of the reinforcement N/mm², and z is the internal lever arm in mm, approximated as $z \approx 0.9 d$.

The reinforcement ratio constraint is then

$$A_s < 0.02 W d \quad (10)$$

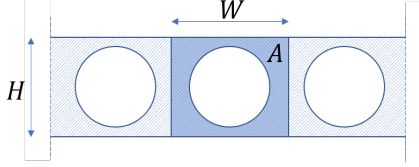


Figure 3: Illustration of a single sphere unit.

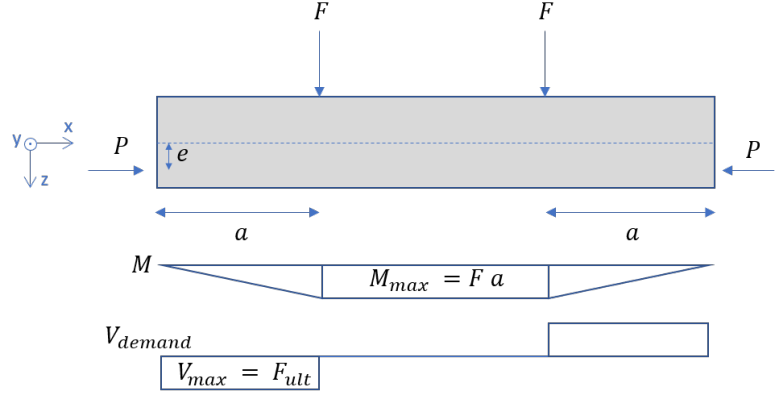


Figure 4: Illustration of the 4-point bending scenario.

2.1.1. Geometrical constraints

Based on the 3-section approach, basic geometrical constraints can be derived. These include : 1. The sum of the spheres' diameters should not exceed the length of the beam, i.e.

$$\sum_{i=1}^N 2r_i \leq L, \quad (11)$$

2. The maximum diameter of the spheres should not exceed the height of the beam, i.e.

$$2 \max r_i \leq H, \quad (12)$$

3. The maximum diameter of the spheres should not exceed the width of the beam, i.e.

$$2 \max r_i \leq W, \quad (13)$$

where N is the number of spheres, r_i is the radius of each sphere, L is the length of the beam, H is the height of the beam, and W is the width of the beam.

3. Problem formulation

The objective of optimization is to minimize the total volume of concrete in a prestressed concrete beam by embedding hollow spheres, while satisfying structural and geometric constraints.

A direct physical parameterization is defined by the sphere radii and longitudinal positions, together with the prestressing force:

$$\mathbf{y} = (r_1, \dots, r_N, x_1, \dots, x_N, P), \quad (14)$$

where N is the number of spheres, r_i is the radius of sphere i , x_i is its center position along the beam axis and P is the applied prestress force.

The objective is the relative concrete volume

$$\min_{\mathbf{y}} V_{\text{rel}}(\mathbf{r}) = \frac{V_{\text{beam}} - V_{\text{void}}}{V_{\text{beam}}}, \quad (15)$$

with

$$V_{\text{beam}} = WHL, \quad V_{\text{void}} = \sum_{i=1}^N \frac{4}{3} \pi r_i^3, \quad (16)$$

where W , H , and L denote the width, height, and length of the beam.

The design is subject to nonlinear and non-convex constraints, including structural requirements (shear and bending capacity, maximum tensile stress under service loads, and reinforcement area limitations), as well as geometric feasibility requirements ensuring that the spheres remain fully contained within the beam boundaries and that no sphere overlap occurs.

4. Mathematical formulation

4.1. Latent-space parameterization

A latent-space parameterization is introduced in order to avoid using explicit bound and geometry handling in the optimizer. The mapping from the latent vector to physical variables enforces bounded radii, admissible prestress values, and a well-defined spacing of spheres by construction. The remaining feasibility conditions are enforced via penalty terms.

The latent vector is defined as

$$u = (s, t, p) \in R^{2N_{\max}+1}, \quad (17)$$

where s encodes the sphere radii, t the weights which control the sphere placement and p the applied prestress force.

4.2. Piecewise smooth mapping to physical variables

In addition, the following piecewise smooth mapping is defined, where $N_{\max}$ is the maximum number of spheres allowed :

$$\begin{aligned} \Phi : R^{2N_{\max}+1} &\longrightarrow R^{N_{\max}} \times R^{N_{\max}} \times R \\ u &\mapsto \Phi(u) = (r(u), x(u), P(u)) \end{aligned} \quad (18)$$

More specifically, the radii encoded by the latent variables s_i are obtained using sigmoid mappings, defined by

$$r_i(u) = R_{\min} + (R_{\max} - R_{\min}) \frac{1}{1 + e^{-s_i}}, \quad (19)$$

where $R_{\min}$ and $R_{\max}$ are the lower and upper bounds on the sphere radii. In addition, spheres with radii below a manufacturability threshold $r_{\text{tol}} > 0$ are treated as absent by setting $r_i = 0$ whenever $r_i < r_{\text{tol}}$.

Let $\epsilon > 0$ be the minimum gap. Softmax-type weights are defined for the $N_{\max} + 1$ gaps (left clearance, surface-to-surface gaps, right clearance) by

$$\begin{aligned} w_i &= \frac{e^{t_i}}{1 + \sum_{j=1}^{N_{\max}} e^{t_j}}, \quad i = 1, \dots, N_{\max}, \\ w_{N_{\max}+1} &= \frac{1}{1 + \sum_{j=1}^{N_{\max}} e^{t_j}}, \end{aligned} \quad (20)$$

so that $\sum_{i=1}^{N_{\max}+1} w_i = 1$. The remaining length after placing all diameters and minimum gaps is

$$L_{\text{rem}} = L - \sum_{i=1}^{N_{\max}} 2r_i - (N_{\max} + 1)\epsilon, \quad (21)$$

which gives the following gaps :

$$g_i = \epsilon + w_i \max(L_{\text{rem}}, 0), \quad i = 1, \dots, N_{\max} + 1. \quad (22)$$

Sphere centers are then placed cumulatively,

$$x_1 = g_1 + r_1, \quad x_i = x_{i-1} + r_{i-1} + g_i + r_i, \quad i = 2, \dots, N_{\max}. \quad (23)$$

By construction $g_i \geq \epsilon$, so overlaps cannot occur; if $L_{\text{rem}} < 0$ the layout overfills the beam and is handled by the end-of-beam penalty.

Finally, the prestressing force is mapped from the latent scalar p via a sigmoid,

$$P(u) = P_{\min} + (P_{\max} - P_{\min}) \frac{1}{1 + e^{-p}}, \quad (24)$$

which guarantees $P(u) \in [P_{\min}, P_{\max}]$ by construction.

4.3. Constraints

All mechanical and geometric requirements are encoded in piecewise smooth functions $c_i(u) \geq 0$, $i = 1, \dots, m$, namely :

- Shear capacity: $c_{\text{shear}}(r_i, x_i) = F_{\text{shear}}(r_i) - F_{\text{demand}}(x_i)$,
- Reinforcement ratio: $c_{\text{bend}}(r_i, x_i, P) = A_{s,\max} - A_s(r_i, x_i, P)$,
- Tensile stress: $c_{\text{tensile}}(r_i, x_i, P) = f_{ctm} - \sigma_t(r_i, x_i, P)$,
- End-of-beam: $c_{\text{end}}(u) = L - (x_{N_{\max}} + r_{N_{\max}})$,
- Minimum gaps: $c_{\text{gap}}(u) = x_{i+1} - x_i - (r_i + r_{i+1}) - \epsilon$.

4.4. Objective function and penalty function

The objective function is defined as :

$$\mathcal{L}(u) = V_{\text{rel}}(r(u)) + \text{Penalty}(\Phi(u)), \quad (25)$$

where

$$V_{\text{rel}}(r(u)) = \frac{WHL - \sum_i \frac{4}{3}\pi r_i(u)^3}{WHL}, \quad (26)$$

is the relative volume of concrete in the entire beam, with W the width of the beam, H the height of the beam and L the length of the beam.

The *Penalty* function is defined as follows, with $\alpha, \beta_1, \beta_2, \beta_3$ given penalty weight factors :

$$\begin{aligned} \text{Penalty}(\Phi(u)) = & \beta_1 \sum_{i=1}^{N_{\max}} \max(F_{\text{demand}}(x_i) - F_{\text{shear}}(r_i), 0) \\ & + \beta_2 \sum_{i=1}^{N_{\max}} \max(A_s(r_i, x_i, P) - A_{s,\max}, 0) \\ & + \beta_3 \sum_{i=1}^{N_{\max}} \max(\sigma_t(r_i, x_i, P) - f_{ctm}, 0) \\ & + \alpha \max(x_{N_{\max}} + r_{N_{\max}} - L, 0) \\ & + \alpha \sum_{i=1}^{N_{\max}-1} \max(\epsilon - (x_{i+1} - x_i - (r_i + r_{i+1})), 0) \end{aligned} \quad (27)$$

4.5. Search space geometry

The latent vector $u \in R^{2N_{\max}+1}$ lies in a smooth Euclidean space of dimension $d = 2N_{\max} + 1$, i.e. a flat differentiable manifold. The feasible region $\mathcal{F}$ defined by :

$$\mathcal{F} = \{u : c_i(u) \geq 0, i = 1, \dots, m\} \quad (28)$$

is defined by piecewise-defined inequality constraints.

However, in the objective function $\mathcal{L}$, the *Penalty* terms are in general non-smooth due to the presence of the *max* functions and the piecewise definition of $F_{demand}(x)$.

Because the constraints are piecewise real-analytic with finitely many regions, the feasible set $\mathcal{F}$ is subanalytic and therefore admits a Whitney stratification into smooth manifolds of varying dimension [10]. The penalized objective $\mathcal{L}$ is smooth within each region but can lose differentiability and may be discontinuous across region boundaries introduced by the max-penalties and the piecewise definition. This nonsmooth landscape motivates a derivative-free method such as CMA-ES.

The resulting optimization problem

$$\min_{u \in R^{2N_{\max}+1}} \mathcal{L}(u),$$

is solved using a CMA-ES algorithm. This approach ensures that every iterate $\Phi(u)$ remains on the implicitly bounded design domain and the remaining constraints are enforced via penalties.

5. Optimization strategy

The CMA-ES algorithm is used with the following settings :

- **Population size and generations** : a maximum of 400 generations with population size of 200 individuals are run.
- **Initial step size** : The search is initialized with a standard deviation $\sigma = 0.5$ in each latent dimension in order to cover the design space broadly.
- **Annealing with adaptive penalty** : In equation 27, we set β_1, β_2 and $\beta_3 = 1$. This way, the mechanical penalty terms keep the formulation minimal, since they directly penalize limit-state violations and vanish at feasibility. Only the geometric penalties are scaled via an annealed factor α_k , because they involve the very small tolerance ϵ and would otherwise be under-weighted numerically. The factor is defined by :

$$\begin{aligned} \alpha_k &= \min(\alpha_{\max}, \alpha_0 (1 + \gamma)^k), \\ \alpha_0 &= 10^2, \quad \gamma = 0.05, \quad \alpha_{\max} = 10^5. \end{aligned} \quad (29)$$

This encourages early generations to focus rather on reducing volume with less feasibility pressure, while later generations have sharp penalties when infeasible.

- **Covariance adaptation** : CMA-ES is run in two phases :
 - (i) In diagonal covariance mode for the first 200 generations to quickly learn an individual step-size for each latent coordinate (i.e. estimate per-dimension scales independently).
 - (ii) In full covariance mode afterwards to exploit correlations between variables.

Table 1: Comparison of optimization methods (mean $\pm$ std over 10 runs).

Method	Best V_{rel}	Feas. rate	Eval.	Time (s)
CMA-ES	0.5068 ± 0.0009	100%	69 620	7.35
DE	0.5571 ± 0.0003	100%	70 200	6.74
PSO	0.5729 ± 0.0324	100%	70 200	4.81
Adam only	0.6926 ± 0.0488	90%	23 501	2.01
BO + Adam	0.7220 ± 0.1324	100%	14 151	15.93

6. Benchmark results

Five optimization strategies are compared on the same beam scenario : cross-section $W \times H = 100 \text{ mm} \times 100 \text{ mm}$, length $L = 1100 \text{ mm}$, up to $N_{\max} = 15$ spheres with radii $r_i \in [0 \text{ mm}, 50 \text{ mm}]$, threshold below which spheres are deactivated $r_{tol} = 20 \text{ mm}$ (for manufacturability reasons), and minimum gap requirement $\epsilon = 1 \times 10^{-5} \text{ mm}$. The 4-point bending setup partitions the beam into three regions of 450 mm–200 mm–450 mm.

- **CMA-ES** with diagonal then full covariance (population size $\lambda = 200$, with a max of 400 generations, terminating earlier in practice, see Tab. 1).
- **Baseline Adam optimization** on 500 iterations with learning rate 10^{-2} , decayed.
- **Bayesian Optimization + Adam**: A Gaussian-process Bayesian Optimization (BO) loop is run for 50 BO calls, then the best suggestion is refined via 300 iterations of Adam on the latent space.
- **Differential Evolution (DE)** with population size $\lambda = 200$ and 350 generations.
- **Particle Swarm Optimization (PSO)**: global-best PSO with $N_p = 200$ particles and 350 iterations.

Each method is evaluated over 10 independent runs. Stochastic methods (CMA-ES, DE, PSO, BO+Adam) are repeated with 10 random seeds (affecting sampling and initialization). Adam, which is deterministic given an initialization, is run from 10 random initializations.

6.1. Performance metrics

The following metrics are measured:

- **Best feasible objective** after all evaluations, $\min_{u: c_i(u) \geq 0} V_{rel}(r(u))$.
- **Convergence speed**: number of evaluations.
- **Feasibility rate**: percentage of runs ending in a fully feasible design ($c_i(u) \geq 0 \forall i$).
- **Runtime**: time per run.

6.2. Results and discussion

Tab. 1 reports performance over 10 independent runs for each optimization strategy. CMA-ES clearly achieves the best solution, reaching a mean objective value of 0.5068 ± 0.0009 (i.e. approximately 50% concrete mass reduction), which is significantly lower than both Adam (0.6926 ± 0.0488) and BO+Adam (0.7220 ± 0.1324). BO+Adam is clearly not suited for the high-dimensionality of the problem. Among the population-based baselines, higher objective values are reached (0.5571 ± 0.0003 for DE and

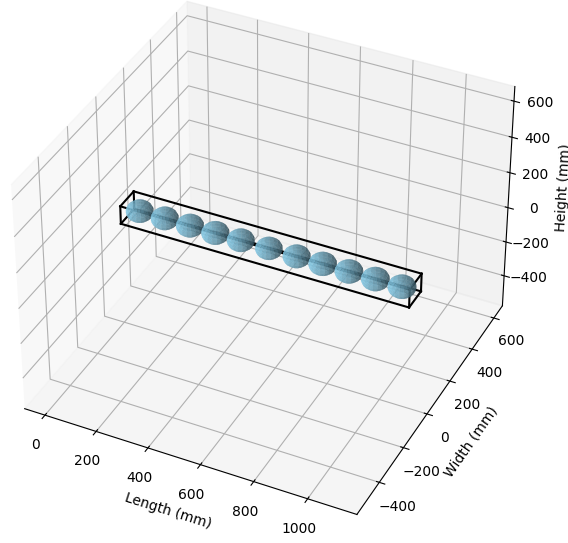


Figure 5: Visualization of the optimal parameters of the hollow spheres in a post-tensioned concrete beam for a given scenario.

0.5729 ± 0.0324 for PSO). The standard deviation across runs is also quite small for CMA-ES and DE, showing that these methods perform robustly, in comparison with the PSO, Adam and BO+Adam methods which converge less consistently.

In addition, all methods except Adam reach feasible designs in every run, whereas Adam only achieves a 90% feasibility rate. The methods also differ significantly in computational behavior. Adam is the fastest in terms of time, but it converges to worse designs, which shows its tendency to become trapped in local minima. The hybrid BO+Adam approach has the highest runtime due to Gaussian-process model updates, but it performs poorly in terms of solution quality. The Eval. column corresponds to the number of calls of the penalized objective $\mathcal{L}$. Evaluation budgets differ across methods due to their standard operating settings. For this metric, CMA-ES, DE and PSO require more function evaluations, as expected for an evolutionary strategy, but remain computationally efficient overall and deliver better results over the gradient-based approaches.

These results indicate that evolutionary, population-based search is well suited to the constrained and non-convex beam design landscape. In particular, the CMA-ES strategy provides the most reliable and highest-quality optimization performance among the tested methods.

Figure 5 illustrates the optimized hollow-sphere configuration for the previously mentioned beam scenario. In the results, the hollow spheres are placed at optimized center positions, with active center-to-center spacings ranging from 96.4 mm to 117.8 mm. The radii are approximately 49.29 mm, with some minimal variations. The prestressing force of 10000 N satisfies the constraints.

7. Conclusion

In this paper, a latent-space formulation for the design optimization of functionally graded concrete beams is presented. The proposed parameterization maps an unconstrained latent vector to bounded radii, admissible prestress values and a well-defined spacing of spheres by construction. Remaining mechanical and geometric requirements are enforced via penalty terms. This avoids explicit bound handling and enables better robustness during the search.

A benchmark is run on a 4-point bending scenario. It demonstrates that CMA-ES provides the best optimization performance among all five tested methods. It reaches approximately 50% concrete mass reduction while satisfying all shear, tensile stress, reinforcement and geometric feasibility constraints.

Future work includes validation on physical beam samples. Initial laboratory tests have already been conducted and confirm the optimized design results observed in simulation. An experimental study will be reported in subsequent work.

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